

# $Q^2$ -Dependence of the Drell-Hearn-Gerasimov integral

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**Abstract.** The energy and four-momentum ( $Q^2$ ) dependence of the photo-absorption cross section on the proton is calculated for helicity  $-\frac{1}{2}$  and  $-\frac{3}{2}$  states. An effective Lagrangian model is used, formulated in terms of meson and baryon degrees of freedom, which obeys crossing symmetry, unitarity, Lorentz and gauge invariance. The difference in the cross sections for the two helicity states, the Drell-Hearn-Gerasimov integral  $I_{DHG}(Q^2)$ , is evaluated at different  $Q^2$ . We find that at small momentum transfer the absolute value of  $I_{DHG}(Q^2)$  first increases to reach a maximum at  $Q^2 \approx 0.05 \text{ GeV}^2$  before decreasing at higher  $Q^2$ .

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## 1 Introduction

Absorption of virtual photons on the nucleon at very high photon four-momentum ( $Q^2$ ) has been proposed as a means to measure the spin content of the nucleon. Since helicity is conserved in the scaling limit, the cross-section difference between parallel and anti-parallel helicities for the photon and the proton should be a measure of the spin carried by the quarks in the proton. This difference, integrated over energy, is called the DHG integral. At large  $Q^2$  this integral has been expressed in terms of a sumrule by Ellis and Jaffe [1] (EJ). Data at high  $Q^2$  seem to agree with the momentum dependence predicted by this QCD-based sumrule. The magnitude is however considerably smaller than predicted and this discrepancy has been known as the “spin crisis”. For real photons,  $Q^2 = 0$ , another, rigorous sumrule has been formulated for this integral by Drell and Hearn [2] and independently by Gerasimov [3] (DHG). While the values predicted by EJ are positive, the DHG value is large and negative and it is an intriguing problem to reconcile the two.

The different values for the sumrule at low and high  $Q^2$  are related to the transition from physics dominated by nucleon resonances [4] (non-perturbative QCD) to the perturbative QCD regime. Several studies [5–8] have emphasized the explicit role played by nucleon resonances in the transitional regime around  $Q^2 \approx 1 \text{ GeV}^2$ . Recently the integral has been calculated in a unitarized isobar model [9] in which a parametrization of resonances, based on a

Breit-Wigner shape, has been used. At the lowest momentum transfers this has been investigated in chiral perturbation theory [10].

The derivation of the sumrules is based on Lorentz and gauge invariance, crossing symmetry, unitarity and causality. It is therefore of interest to investigate the sumrule in a model in which most of these symmetries are obeyed. We present a calculation of the strength distribution for  $Q^2 \leq 1 \text{ GeV}^2$  in the model developed in [11,12]. This model obeys crossing symmetry, unitarity, Lorentz and gauge invariance. It is formulated in terms of meson and nucleon degrees of freedom which includes nucleon resonances in an effective-Lagrangian formalism.

## 2 Outline of the model

We use the relativistic effective-Lagrangian formalism presented in [11,12]. The model is based on the K-matrix approach. The kernel is constructed from the direct (s), exchange (u) and meson exchange (t-channel) tree-level amplitudes. In the s- and u-channels all spin  $-1/2$  and  $-3/2$  baryon resonances with masses below 1.7 GeV are included. The use of the K-matrix approach guarantees unitarity in the coupled-channel  $(\gamma + N) \otimes (\pi + N)$  space. Observing unitarity is of crucial importance for the calculation of cross sections for photon energies exceeding 250 MeV. Coupling to channels outside this model space is included in an approximate manner through the introduction of an imaginary part in the self-energy of the s-channel resonances [12]. The coupling parameters have been obtained from a simultaneous fit to pion-nucleon

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phase shifts, pion-photoproduction multipoles and cross sections for Compton scattering [12]. The model is Lorentz and gauge invariant and obeys crossing symmetry. The chiral-symmetry constraints are also respected since the low-energy  $\pi N$  scattering is well described. Here we will only mention the details which are of interest for the present application.

Of particular importance for the DHG integral is the treatment of the  $\Delta$ -resonance. The most general  $\gamma N \Delta$  vertex for finite  $q^2 = -Q^2$  is given by [13, 14]

$$\begin{aligned} \Gamma_{N\gamma\rightarrow\Delta^\alpha} &= \frac{i}{2M} F_{VMD}(q^2) [G_1 \theta_{\alpha\beta}(z_1) \gamma_\delta \\ &\quad - \frac{G_2}{2M} \theta_{\alpha\beta}(z_2) p_\delta \\ &\quad - \frac{G_3}{2M} \theta_{\alpha\beta}(z_3) q_\delta] \gamma_5 (q^\beta \varepsilon^\delta - q^\delta \varepsilon^\beta) \quad (1) \\ \theta_{\alpha\beta}(a_i) &= g_{\alpha\beta} + a_i \gamma_\alpha \gamma_\beta \quad \text{and} \\ a_i &\equiv -(z_i + 1/2) \quad \text{for } i = 1, 2, 3 \end{aligned}$$

where  $G_i = g_i T_3$  and  $T_3$  is the  $N \leftrightarrow \Delta$  isospin transition operator. The constants  $g_1$ ,  $g_2$  and off-shell parameters  $z_1$ ,  $z_2$  are fixed from the fit [12] for the real photons. The constant  $g_3$  (and  $z_3$ ) does not contribute for  $Q^2 = 0$  and therefore we will choose  $g_3 = 0$  in the present investigation. In general, at finite  $Q^2$  the coupling  $g_3$  affects the longitudinal multipole  $L_{1+}$  as well as the ratio  $E_{1+}/M_{1+}$  for the  $\Delta$ -resonance. The transition form factor is taken in the form

$$F_{VMD}(q^2) = \frac{2m_\rho^4}{(2m_\rho^2 - q^2)(m_\rho^2 - q^2)}, \quad (2)$$

as inspired by vector-meson dominance where  $m_\rho$  is the  $\rho$ -meson mass.

The DHG integral at finite  $Q^2$  can be introduced as

$$\begin{aligned} I_{DHG}(Q^2) &= \frac{2M^2}{Q^2} \int_0^1 dx \left( g_1(x, Q^2) - \frac{4x^2 M^2}{Q^2} g_2(x, Q^2) \right) \\ &= \frac{M^2}{4\pi^2 \alpha} \int_{Q^2/2M}^\infty \frac{d\nu}{\nu} \sigma^{TT} \quad (3) \end{aligned}$$

which relates this sumrule directly to the transverse-transverse interference cross section

$$\sigma^{TT} = \frac{1}{2} (\sigma_{1/2}^T - \sigma_{3/2}^T) \quad (4)$$

for inelastic electron scattering on the nucleon. Throughout this paper we use the Bjorken variable  $x = Q^2/2M\nu$  and  $\nu = p \cdot q/M$ , the energy of the virtual photon in the lab system. The total absorption cross section for a transverse virtual photon in a state with total helicity  $\lambda$  is denoted by  $\sigma_\lambda^T$  where the dependence on  $\nu$  and  $Q^2$  is not indicated for ease of writing.

The spin-dependent structure functions which enter in (3) are defined as

$$g_1(x, Q^2) = \frac{M\nu}{4\pi^2 \alpha (1 + Q^2/\nu^2)} \left( \sigma^{TT} + \frac{\sqrt{Q^2}}{\nu} \sigma_{1/2}^{LT} \right), \quad (5)$$

$$g_2(x, Q^2) = \frac{M\nu}{4\pi^2 \alpha (1 + Q^2/\nu^2)} \left( -\sigma^{TT} + \frac{\nu}{\sqrt{Q^2}} \sigma_{1/2}^{LT} \right), \quad (6)$$

where  $\sigma_{1/2}^{TL}$  is the transverse-longitudinal interference cross section, suppressing again the energy and momentum dependence. Note that structure functions  $G_{1,2}$  introduced by Bjorken [15] are related to  $g_{1,2}(x, Q^2)$  through

$$\begin{aligned} M^2 \nu G_1(\nu, Q^2) &= g_1(x, Q^2), \\ M \nu^2 G_2(\nu, Q^2) &= g_2(x, Q^2). \end{aligned}$$

It should be noted that at finite  $Q^2$  (3) differs from both [7] and [8]. In particular, in [8] the DHG integral in addition to  $\sigma^{TT}$  contains also  $\sigma_{1/2}^{LT}$  contribution. Our definitions agree with [16] and have been chosen since they yield the expression for the DHG integral as measured in recent experiments. In the limit of real photon or in the scaling limit,  $(Q^2, \nu) \rightarrow \infty$  at fixed  $x = Q^2/2M\nu$ , above differences vanish.

At the photon point [2, 3]

$$I_{DHG}(Q^2 = 0) = -\frac{1}{4} \kappa^2 \quad (7)$$

with  $\kappa$  being the anomalous magnetic moment of the nucleon, and in the scaling regime

$$I_{DHG}(Q^2) \rightarrow \frac{2M^2}{Q^2} \int_0^1 g_1(x) dx = \frac{2M^2}{Q^2} \Gamma_1, \quad (8)$$

where  $\Gamma_1$  is the moment of  $g_1$ . Experiment gives for the proton  $\Gamma_1^p \approx 0.126$  at  $Q^2 = 10.7 \text{ GeV}^2$  [17] while the prediction of EJ [1] is  $\Gamma_1^p = 0.185$ .

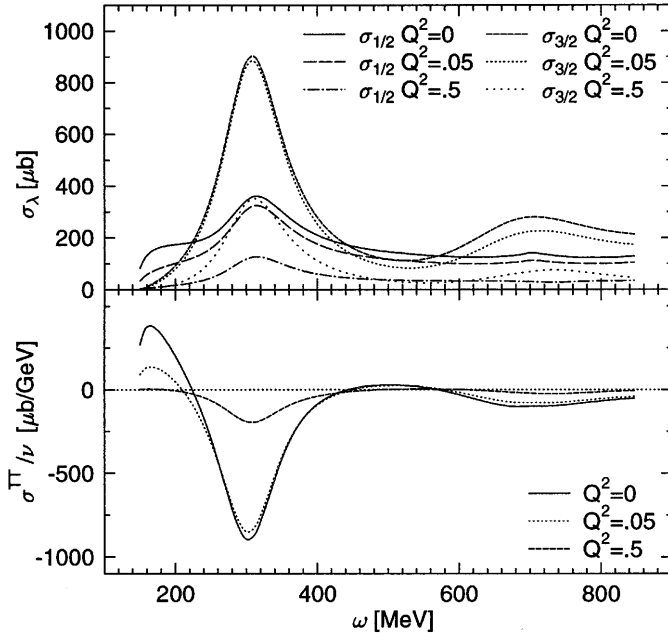
### 3 Results

The total photo-nucleon cross section can be calculated from the imaginary part of the forward-scattering  $\gamma^* N \rightarrow \gamma^* N$  amplitude for total helicity- $\frac{3}{2}$  and  $-\frac{1}{2}$  states. In Fig. (1) the cross sections for the two initial helicity states are plotted versus energy  $\omega = (s - M^2)/2M$  where  $\sqrt{s}$  is the invariant energy of the system. The energy  $\omega$  is related to the integration variable in (3) through

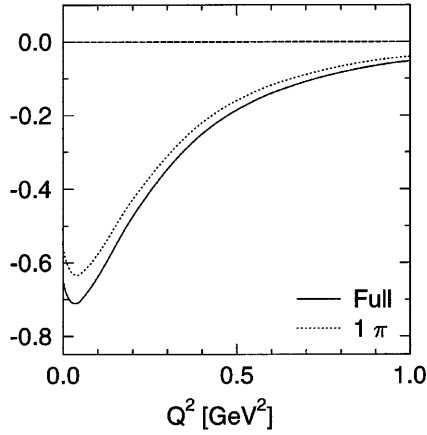
$$\omega = \nu - Q^2/2M \quad (9)$$

and has the advantage that the s-channel resonances occur at a value of  $\omega$ , independent of  $Q^2$ . The large peak in the cross section at  $\omega \approx 300 \text{ MeV}$  is due to the  $\Delta$ -resonance while the one at  $\omega \approx 700 \text{ MeV}$  is due to the  $D_{13}$ -resonance.

At energies below the  $\Delta$ -resonance the pion-photon seagull term, which is needed to ensure gauge invariance for the pion-electroproduction amplitude, gives by far the dominant contribution. It contributes to the helicity- $\frac{1}{2}$  states only and thus it gives a sizable positive contribution to the DHG integral at  $Q^2=0$ . Since this contribution to the cross section is inversely proportional to the momentum of the virtual photon, it strongly diminishes when  $\sqrt{Q^2} \approx \nu$  causing the decrease (increase of the absolute



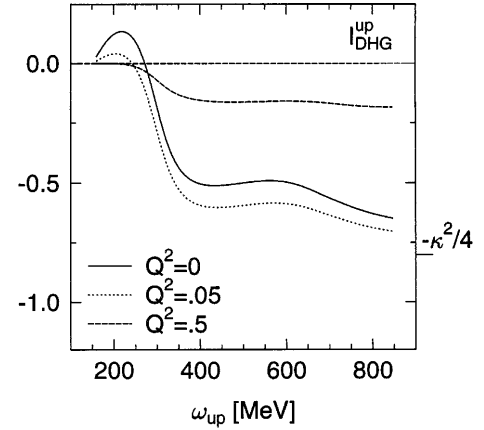
**Fig. 1.** Energy dependence of the photo-absorption cross section for parallel ( $\sigma_{3/2}^T$ ) and anti-parallel ( $\sigma_{1/2}^T$ ) photon and nucleon helicities at different momentum transfer  $Q^2$  as indicated in the figure. The lower panel shows the energy dependence of the integrand in (3).  $Q^2$  is indicated in  $\text{GeV}^2$



**Fig. 2.** The momentum dependence of the DHG integral. The dotted line shows the contribution of the single-pion production channel

magnitude) of the DHG integral seen in Fig. (2) at low  $Q^2$ . The dominant contribution to the DHG integral originates from the  $\Delta$ -resonance and is negative in sign. Only at values of  $Q^2$  of the order of the  $\rho$ -meson mass the form factor (2) starts to cut this  $\Delta$ -contribution giving rise to a general decrease of the absolute magnitude of the DHG integral seen in Fig. (2). With increasing  $Q^2$  the absolute value of  $I_{DHG}(Q^2)$  thus first increases to reach a maximum at  $Q^2=0.05 \text{ GeV}^2$  after which it strongly decreases.

The above features of the DHG integral can also be observed from Fig. (3) where the dependence of the DHG



**Fig. 3.** Dependence of the integral defined in (10) on the upper integration limit for different values of  $Q^2$  (in  $\text{GeV}^2$ ). The sumrule value  $-\kappa^2/4$  is indicated on the right

integral on the upper integration limit

$$I_{DHG}^{up}(Q^2) = \frac{M^2}{4\pi^2\alpha} \int_{Q^2/2M}^{\nu_{up}} \frac{d\nu}{\nu} \sigma^{TT} \quad (10)$$

is given as function of  $\omega_{up} = \nu_{up} - Q^2/2M$  (see (9)). It can be seen that the important contribution to this integral is generated in the region of the  $\Delta$ -resonance. At  $\omega_{max} \approx 800 \text{ MeV}$ , the maximum energy where we can apply the present model with confidence, about 80% of the DHG sumrule value (at  $Q^2 = 0$ ) is reached.

In Fig. (2) the single-pion production contribution to  $I_{DHG}(Q^2)$  is shown also. At small  $Q^2$  it contributes 86% to the integral, this fraction is somewhat larger than 76% as was found by Karliner [18] at the real-photon point. At higher  $Q^2$  the multiple-pion emission contribution decreases in absolute magnitude but increases in relative importance to 23% at  $Q^2=1 \text{ GeV}^2$ .

Finally, as we mentioned, the coupling parameter  $g_3$  in (1) has been chosen zero. In general, at finite  $Q^2$  the DHG integral depends on its value. We have checked that for a moderate positive value,  $g_3 \approx |g_1|$ , the DHG integral changes sign and becomes positive at  $Q^2 \approx 1 \text{ GeV}^2$ . The effect of this coupling on the DHG integral and pion electro-production multipoles will be studied in detail in a separate publication.

## 4 Conclusions

Our calculations in an effective-Lagrangian model show that at  $Q^2 < 1 \text{ GeV}^2$  the DHG integral is dominated by resonance contributions, mainly by the  $P_{33}$  and the  $D_{13}$ -resonances. Due to the form factors which are implemented in the calculation these contributions decrease rather sharply at moderate  $Q^2$ . This is similar to the results presented in [9] even though the models are different. At large  $Q^2$  exceeding a few  $\text{GeV}^2$  the present model, being based on nucleon and meson degrees of freedom, loses validity since the quark structure of the particles starts

to play an increasingly important role. In the framework of an effective-Lagrangian approach one could model the quark structure by ascribing different  $Q^2$ -dependences to electric and magnetic couplings of resonances. This might well explain the change of sign of the integral at large  $Q^2$ .

At small  $Q^2$ ,  $Q^2 < 0.05 \text{ GeV}^2$ , we observe a striking increase in the absolute magnitude of the DHG integral. This is due to the particular momentum dependence of the seagull (or  $\gamma\pi NN$ ) contribution to the dominant pion-electroproduction process. Surprisingly this behavior of  $I_{DHG}(Q^2)$  is opposite to the dependence obtained in the ChPT calculations [10].

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